OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE **4723/01**

MATHEMATICS

Core Mathematics 3

MONDAY 2 JUNE 2008 Morning

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

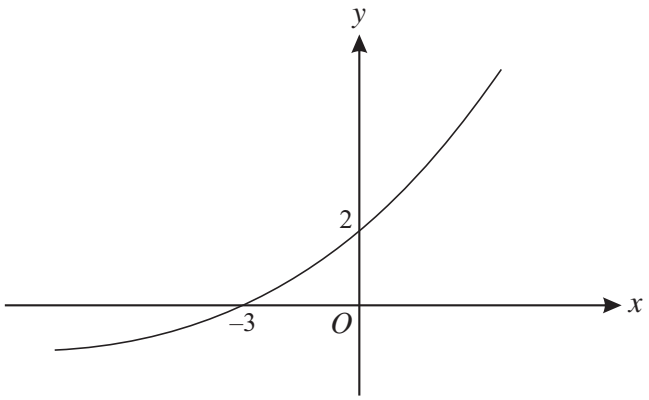
- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

[Turn over

1 Find the exact solutions of the equation $|4x - 5| = |3x - 5|$. [4]

2



The diagram shows the graph of $y = \mathrm{f}(x)$. It is given that $\mathrm{f}(-3) = 0$ and $\mathrm{f}(0) = 2$. Sketch, on separate diagrams, the following graphs, indicating in each case the coordinates of the points where the graph crosses the axes:

(i) $y = \mathrm{f}^{-1}(x)$, [2]

(ii) $y = -2\mathrm{f}(x)$. [3]

3 Find, in the form $y = mx + c$, the equation of the tangent to the curve

$$y = x^2 \ln x$$

at the point with x-coordinate e. [6]

4 The gradient of the curve $y = (2x^2 + 9)^{\frac{5}{2}}$ at the point P is 100.

(i) Show that the x-coordinate of P satisfies the equation $x = 10(2x^2 + 9)^{-\frac{3}{2}}$. [3]

(ii) Show by calculation that the x-coordinate of P lies between 0.3 and 0.4. [3]

(iii) Use an iterative formula, based on the equation in part **(i)**, to find the x-coordinate of P correct to 4 decimal places. You should show the result of each iteration. [3]

5 **(a)** Express $\tan 2\alpha$ in terms of $\tan \alpha$ and hence solve, for $0° < \alpha < 180°$, the equation

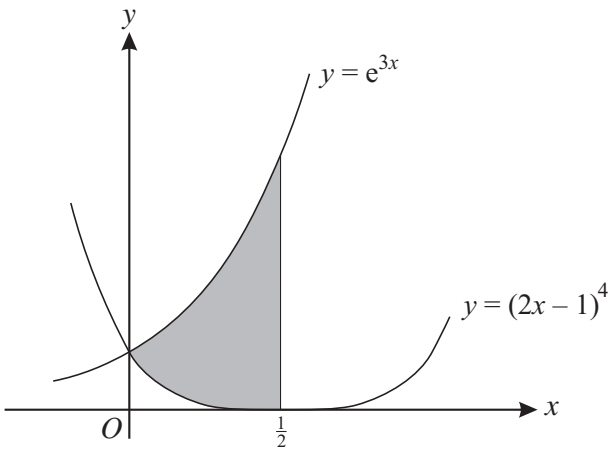
$$\tan 2\alpha \tan \alpha = 8.$$ [6]

(b) Given that β is the acute angle such that $\sin \beta = \frac{6}{7}$, find the exact value of

(i) $\operatorname{cosec} \beta$, [1]

(ii) $\cot^2 \beta$. [2]

6



The diagram shows the curves $y = e^{3x}$ and $y = (2x - 1)^4$. The shaded region is bounded by the two curves and the line $x = \frac{1}{2}$. The shaded region is rotated completely about the x-axis. Find the exact volume of the solid produced. [9]

7 It is claimed that the number of plants of a certain species in a particular locality is doubling every 9 years. The number of plants now is 42. The number of plants is treated as a continuous variable and is denoted by N. The number of years from now is denoted by t.

(i) Two equivalent expressions giving N in terms of t are

$$N = A \times 2^{kt} \qquad \text{and} \qquad N = Ae^{mt}.$$

Determine the value of each of the constants A, k and m. [4]

(ii) Find the value of t for which $N = 100$, giving your answer correct to 3 significant figures. [2]

(iii) Find the rate at which the number of plants will be increasing at a time 35 years from now. [3]

8 The expression $\mathrm{T}(\theta)$ is defined for θ in degrees by

$$\mathrm{T}(\theta) = 3\cos(\theta - 60^\circ) + 2\cos(\theta + 60^\circ).$$

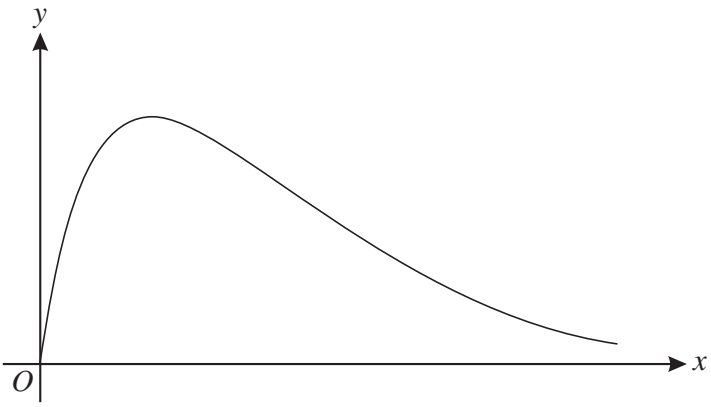
(i) Express $\mathrm{T}(\theta)$ in the form $A\sin\theta + B\cos\theta$, giving the exact values of the constants A and B. [3]

(ii) Hence express $\mathrm{T}(\theta)$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [3]

(iii) Find the smallest positive value of θ such that $\mathrm{T}(\theta) + 1 = 0$. [4]

[Question 9 is printed overleaf.]

9



The function f is defined for the domain $x \geqslant 0$ by

$$\mathrm{f}(x) = \frac{15x}{x^2 + 5}.$$

The diagram shows the curve with equation $y = \mathrm{f}(x)$.

(i) Find the range of f. [6]

(ii) The function g is defined for the domain $x \geqslant k$ by

$$\mathrm{g}(x) = \frac{15x}{x^2 + 5}.$$

Given that g is a one-one function, state the least possible value of k. [1]

(iii) Show that there is no point on the curve $y = \mathrm{g}(x)$ at which the gradient is -1. [4]

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1 Either: Obtain $x = 0$ **B1** ignoring errors in working
Form linear equation with signs of $4x$ and $3x$ different **M1** ignoring other sign errors
State $4x - 5 = -3x + 5$ **A1** or equiv without brackets
Obtain $\frac{10}{7}$ and no other non-zero value(s) **A1** or exact equiv
4

Or: Obtain $16x^2 - 40x + 25 = 9x^2 - 30x + 25$ **B1** or equiv
Attempt solution of quadratic equation **M1** at least as far as factorisation or use of formula
Obtain $\frac{10}{7}$ and no other non-zero value(s) **A1** or exact equiv
Obtain 0 **B1** ignoring errors in working
4

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2 (i) Show graph indicating attempt at reflection in $y = x$ **M1** with correct curvature and crossing negative y-axis and positive x-axis
Show correct graph with x-coord 2 and y-coord -3 indicated **A1**
2

(ii) Show graph indicating attempt at reflection in x-axis **M1** with correct curvature and crossing each negative axis
Show correct graph with x-coord -3 indicated **A1**
… and y-coord -4 indicated **A1**
[SC: Incorrect curve earning M0 but both correct intercepts indicated B1]
3

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3 Attempt use of product rule **M1** … + … form
Obtain $2x \ln x + x^2 . \frac{1}{x}$ **A1** or equiv
Substitute e to obtain 3e for gradient **A1** or exact (unsimplified) equiv
Attempt eqn of straight line with numerical gradient **M1** allowing approx values
Obtain $y - e^2 = 3e(x - e)$ **A1√** or equiv; following their gradient provided obtained by diffn attempt; allow approx values
Obtain $y = 3ex - 2e^2$ **A1** in terms of e now and in requested form
6

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4 (i) Differentiate to obtain form $kx(2x^2 + 9)^n$ **M1** any constant k; any $n < \frac{5}{2}$
Obtain correct $10x(2x^2 + 9)^{\frac{3}{2}}$ **A1** or (unsimplified) equiv
Equate to 100 and confirm $x = 10(2x^2 + 9)^{-\frac{3}{2}}$ **A1** **AG**; necessary detail required
3

(ii) Attempt relevant calculations with 0.3 and 0.4 **M1**
Obtain at least one correct value **A1**

x	f(x)	x – f(x)	f′(x)
0.3	0.3595	–0.0595	83.4
0.4	0.3515	0.0485	113.8

Obtain two correct values and conclude appropriately **A1** noting sign change or showing 0.3 < f(0.3) and 0.4 > f(0.4) or showing gradients either side of 100
3

	(iii)	Obtain correct first iterate	**B1**	
		Carry out correct process	**M1**	finding at least 3 iterates in all
		Obtain 0.3553	**A1**	answer required to exactly 4 dp
			3	
		[0.3 → 0.35953 → 0.35497 → 0.35534 → 0.35531; 0.35 → 0.35575 → 0.35528 → 0.35532 (→ 0.35531); 0.4 → 0.35146 → 0.35563 → 0.35529 → 0.35532]		
5	**(a)**	Obtain expression of form $\frac{a\tan\alpha}{b+c\tan^2\alpha}$	**M1**	any non-zero constants a, b, c
		State correct $\frac{2\tan\alpha}{1-\tan^2\alpha}$	**A1**	or equiv
		Attempt to produce polynomial equation in $\tan\alpha$	**M1**	using sound process
		Obtain at least one correct value of $\tan\alpha$	**A1**	$\tan\alpha = \pm\sqrt{\frac{4}{5}}$
		Obtain 41.8	**A1**	allow 42 or greater accuracy; allow 0.73
		Obtain 138.2 and no other values between 0 and 180	**A1**	allow 138 or greater accuracy
		[SC: Answers only 41.8 or … B1; 138.2 or … and no others B1]	**6**	
	(b)(i)	State $\frac{7}{6}$	**B1**	
			1	
	(ii)	Attempt use of identity linking $\cot^2\beta$ and $\text{cosec}^2\beta$	**M1**	or equiv retaining exactness; condone sign errors
		Obtain $\frac{13}{36}$	**A1**	or exact equiv
			2	
6		Integrate k_1e^{nx} to obtain k_2e^{nx}	**M1**	any constants involving π or not; any n
		Obtain correct indefinite integral of their k_1e^{nx}	**A1**	
		Substitute limits to obtain $\frac{1}{6}\pi(e^3-1)$ or $\frac{1}{6}(e^3-1)$	**A1**	or exact equiv perhaps involving e^0
		Integrate $k(2x-1)^n$ to obtain $k'(2x-1)^{n+1}$	**M1**	any constants involving π or not; any n
		Obtain correct indefinite integral of their $k(2x-1)^n$	**A1**	
		Substitute limits to obtain $\frac{1}{18}\pi$ or $\frac{1}{18}$	**A1**	or exact equiv
		Apply formula $\int\pi y^2\text{d}x$ at least once	**B1**	for $y=e^{3x}$ and/or $y=(2x-1)^4$
		Subtract, correct way round, attempts at volumes y^2	**M1**	allow with π missing but must involve
		Obtain $\frac{1}{6}\pi e^3-\frac{2}{9}\pi$	**A1**	or similarly simplified exact equiv
			9	
7	**(i)**	State $A=42$	**B1**	
		State $k=\frac{1}{9}$	**B1**	or 0.11 or greater accuracy
		Attempt correct process for finding m	**M1**	involving logarithms or equiv
		Obtain $\frac{1}{9}\ln 2$ or 0.077	**A1**	or 0.08 or greater accuracy
			4	
	(ii)	Attempt solution for t using either formula	**M1**	using correct process (log'ms or T&I or …)
		Obtain 11.3	**A1**	or greater accuracy; allow 11.3 ± 0.1
			2	
	(iii)	Differentiate to obtain form Be^{mt}	**M1**	where B is different from A
		Obtain $3.235e^{0.077t}$	**A1√**	or equiv; following their A and m
		Obtain 47.9	**A1**	allow 48 or greater accuracy
			3	

8	**(i)**	Show at least correct $\cos\theta\cos 60 + \sin\theta\sin 60$ or $\cos\theta\cos 60 - \sin\theta\sin 60$	**B1**	
		Attempt expansion of both with exact numerical values attempted	**M1**	and with $\cos 60 \neq \sin 60$
		Obtain $\frac{1}{2}\sqrt{3}\sin\theta + \frac{5}{2}\cos\theta$	**A1**	or exact equiv
			3	
	(ii)	Attempt correct process for finding R	**M1**	whether exact or approx
		Attempt recognisable process for finding α	**M1**	allowing sin / cos muddles
		Obtain $\sqrt{7}\sin(\theta + 70.9)$	**A1**	allow 2.65 for R; allow 70.9 ± 0.1 for α
			3	
	(iii)	Attempt correct process to find any value of θ + their α	**M1**	
		Obtain any correct value for $\theta + 70.9$	**A1**	–158, –22, 202, 338, …
		Attempt correct process to find θ + their α in 3rd quadrant	**M1**	or several values including this
		Obtain 131	**A1**	or greater accuracy and no other
		[SC for solutions with no working shown: Correct answer only B4; 131 with other answers B2]		
			4	
9	**(i)**	Attempt use of quotient rule	***M1**	or equiv; allow u / v muddles
		Obtain $\dfrac{75 - 15x^2}{(x^2 + 5)^2}$	**A1**	or (unsimplified) equiv; this **M1A1** available at any stage of question
		Equate attempt at first derivative to zero and rearrange to solvable form	**M1**	dep ***M**
		Obtain $x = \sqrt{5}$ or 2.24	**A1**	or greater accuracy
		Recognise range as values less than y-coord of st pt	**M1**	allowing < here
		Obtain $0 \leq y \leq \frac{3}{2}\sqrt{5}$	**A1**	any notation; with ≤ now; any exact equiv
			6	
	(ii)	State $\sqrt{5}$	**B1**√	following their x-coord of st pt; condone answer $x \geq \sqrt{5}$ but not inequality with k
			1	
	(iii)	Equate attempt at first derivative to –1 and attempt simplification	***M1**	and dependent on first **M** in part **(i)**
		Obtain $x^4 - 5x^2 + 100 = 0$	**A1**	or equiv involving 3 non-zero terms
		Attempt evaluation of discriminant or equiv	**M1**	dep ***M**
		Obtain –375 or equiv and conclude appropriately	**A1**	
			4	